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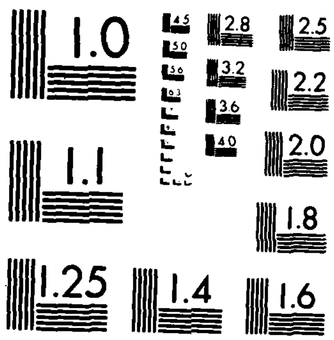
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The focus of this research is the filtering jump processes. To investigate the filtering of manifold-valued processes, their approximation by random walks and Markov chains was studied. The object was to approximate a signal process by a finite-state jump process ofr which a finite-dimensional filter is available. Four papers were published during the past year, including "The existence of smooth densities for the prediction, filtering and smoothing problems" and "The partially observed stochastic minimum principle".

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FILTERING OF JUMP PROCESSES

(Research title changed to 'The existence of smooth densities for the prediction, filtering and smoothing problem' from 01 October 1987.)

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30 October 1987

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WORK COMPLETED

To investigate the filtering of manifold valued processes, their approximation by random walks and Markov chains was considered. In particular, the papers of Gangolli, (Zeits. fur Warsch. 2(1964)), Jorgensen (Zeits, fur Warsch. 32(1964)), and Bismut (Lecture Notes in Math. 866) were studied. I also enjoyed extensive discussions on this topic with Peter Antonelli of the Mathematics Department here at the University of Alberta, and Tom Kurtz, Chairman of the Mathematics Department at the University of Wisconsin. (I was able to talk with Dr. Kurtz at the American Math. Society meeting in San Antonio and again at the meeting on Diffusion Approximations in Austria.) The object was to approximate a signal process by a finite state jump process for which a finite dimensional filter is available.

In principle the above program appears solvable. However, the technicalites were quickly becoming complicated and tedious.

From January to April of this year Dr. Michael Kohlmann, Professor at the University of Konstanz, W. Germany, visited me at the University of Alberta. We have worked together in the past and this time our collaboration was exceptionally productive, resulting in at least nine papers.

In the spring of 1986 I was Distinguished Lecturer at the Systems Research Center, University of Maryland, which is directed by Dr. John Baras. Part of my lectures concerned the optimal control of a partially observed Markov chain and Dr. Baras and I began to discuss whether the techniques I used could be applied to partially observed diffusions or Ito processes. Dr. Kohlmann and I were able to complete this program in [3]. It is shown how a minimum principle, analogous to the Pontrjagin minimum principle, can be obtained by differentiating the statement that a control u^* is optimal. Using backward and forward stochastic flows we explicitly compute the change in the cost due to a 'strong' variation of an optimal control. The only technical difficulty is the justification of the differentiation. The method is conceptually and technically simpler than that employed by Haussmann (S.I.A.M. J. Control and Optimization 25(1987)), and the adjoint, or co-state, process is identified. If the drift coefficient is differentiable in the control variable we show how a minimum principle of Bensoussan, (Stochastics 9 (1983)), follows from our result.

This spring I discovered a short, simple derivation of the integrand when a martingale is represented as a stochastic integral. This result was known, but previous proofs used, for example, perturbations in function space and Girsanov's theorem; our proof just uses the differentiability of solutions of stochastic differential equations and the Ito differentiation rule. When the optimal control of a diffusion is considered, the minimum expected cost from any time onwards is a martingale; this is just a re-statement of the principle of optimality: if you have done your best so far and will do your best from now on, you do not expect the expected value of the minimum cost to vary. Using the representation theorem mentioned above Dr. Kohlmann and I were able to write down the integrand in its representation as a stochastic integral, [4]. This integrand is, in fact, the adjoint process in the stochastic minimum principle and using stochastic calculus we were able to derive the equation satisfied by the adjoint process.

The Malliavin calculus is an infinite dimensional calculus of variations in function space. It is a complicated and sophisticated subject, with connections and applications in many areas, from Hörmander's results on hypoelliptic partial differential operators, to large deviations and the Atiyah-Singer index theory. Using the martingale representation result mentioned above Dr. Kohlmann and I were able to give a simple proof of some of the results of the Malliavin calculus concerning the existence of densities for certain diffusions. These were presented in the invited paper [1] given at the Workshop on Diffusion Approximation, International Institute for Applied Systems Analysis, Austria, in July 1987.

In a paper in the Journal of Functional Analysis 44(1981), Bismut and Michel apply a 'conditional' form of the Malliavin calculus to show the existence of smooth densities for the conditional expectation of the signal in filtering and smoothing problems. However, Bismut and Michel's paper is complicated and hard to read. In [2] Dr. Kohlmann and I develop a 'conditional' form of our simplified treatment of the Malliavin calculus and, under Hörmander's conditions on the coefficient vector fields, show the existence of smooth conditional probability densities in the prediction, filtering and smoothing problems. As noted in the paper, Hörmander's condition is a local condition so the results are true for manifold valued processes.

TRAVEL AND EQUIPMENT

I used some of the travel funds to attend the January 1987 Annual Meeting of the American Mathematical Society in San Antonio.

I also was invited by Professor Kallianpur to speak at the meeting on Diffusion Approximation held at the International Institute for Applied Systems Analysis, Laxenburg, Austria in July 1987. My talk was well received and is written up in [1]. I was also chairman of a session. En route I stopped in England to visit Dr. Kopp in Hull. I have been appointed an Honorary Professor at Hull University in England, and Dr. Kopp and I completed a paper solving Kolmogorov's equations for stochastic flows. The meeting in Austria paid me a per diem allowance, so only a portion of my expenses for the trip were charged to the grant.

As requested in the grant, I have also purchased a Zenith AT, together with a math-coprocessor chip and the Gauss software. The Gauss software is quite fascinating and powerful, particularly for statistical problems. I hope to use these to investigate simulations.

CONCLUSIONS

My work is going exceptionally well. As described above, I have derived results on stochastic control, the Malliavin calculus, and filtering. Possible extensions to be looked at this year include a simplified treatment of the Malliavin calculus for jump processes, a martingale representation and related results for non-Markov processes, and large deviations.

I hope the A.F.O.S.R. is pleased to be associated with what I have done, and I am pleased and grateful the contract has been renewed for a second year.

PUBLICATIONS

1. with M. Kohlmann, Martingale representation and the Malliavin calculus. Invited paper, Workshop on Diffusion Approximation, IIASA, Austria. Submitted to Applied Math. and Optimization.

2. with M. Kohlmann, The existence of smooth densities for the prediction, filtering and smoothing problems. Submitted to Acta Applic. Math.

3. with J. Baras and M. Kohlmann, The partially observed stochastic minimum principle. Submitted to S.I.A.M. Jour. Control and Opt.

4. with Kohlmann, The adjoint process in stochastic optimal control. Submitted to Applied Math. and Optimization.

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